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Implications for CP asymmetries of improved data on $B \to K^0\pi^0$

Michael Gronau[1] and Jonathan L. Rosner

Enrico Fermi Institute and Department of Physics, University of Chicago
Chicago, IL 60637, U.S.A.

The decay $B^0 \to K^0\pi^0$, dominated by a $b \to s$ penguin amplitude, holds the potential for exhibiting new physics in this amplitude. In the pure QCD penguin limit one expects $C_{K\pi} = 0$ and $S_{K\pi} = \sin 2\beta$ for the coefficients of $\cos \Delta mt$ and $\sin \Delta mt$ in the time-dependent CP asymmetry. Small non-penguin contributions lead to corrections to these expressions which are calculated in terms of isospin-related $B \to K\pi$ rates and asymmetries, using information about strong phases from experiment. We study the prospects for incisive tests of the Standard Model through examination of these corrections. We update a prediction $C_{K\pi} = 0.15 \pm 0.04$, pointing out the sensitivity of a prediction $S_{K\pi} \approx 1$ to the measured branching ratio for $B^0 \to K^0\pi^0$ and to other observables.

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One of the most challenging CP asymmetry measurements in B meson decays has involved the coefficients $C_{K\pi}$ and $S_{K\pi}$ in the time-dependent asymmetry measured in $B^0 \to K_S\pi^0$ [1]

$$A(t) = \frac{\Gamma(\overline{B}^0(t) \to \overline{K}^0\pi^0) - \Gamma(B^0(t) \to K^0\pi^0)}{\Gamma(\overline{B}^0(t) \to \overline{K}^0\pi^0) + \Gamma(B^0(t) \to K^0\pi^0)} = -C_{K\pi}\cos(\Delta mt) + S_{K\pi}\sin(\Delta mt) \ . \quad (1)$$

The parameter $C_{K\pi}$ is related to the direct CP asymmetry by $C_{K\pi} \equiv -A_{CP}(B^0 \to K^0\pi^0)$. The decay $B^0 \to K^0\pi^0$ is expected to be dominated by the $b \to s$ penguin amplitude and thus is a good place to look for any new physics that may arise in this amplitude [2–4]. In the pure QCD penguin limit one expects $C_{K\pi} = 0$ and $S_{K\pi} = \sin 2\beta$, respectively, where $\beta = (21.5 \pm 1.0)^\circ$ [5] is an angle in the unitarity triangle. Accounting for small non-penguin contributions leads to corrections to these expressions, which are calculable in terms of isospin-related $B \to K\pi$ decay rates and asymmetries. In this Letter we study the prospects for incisive tests of the Standard Model through examination of these corrections. We update a prediction $C_{K\pi} = 0.15 \pm 0.04$ and point out the sensitivity of a recent theoretical prediction $S_{K\pi} \approx 1$ [6] to the branching ratio for $B^0 \to K^0\pi^0$ and to other observables.

[1]On sabbatical leave from the Physics Department, Technion, Haifa 32000, Israel.

Table I: Measurements of $C_{K\pi}$ and $S_{K\pi}$.

Ref.	$C_{K\pi}$	$S_{K\pi}$
BaBar [7]	$0.24 \pm 0.15 \pm 0.03$	$0.40 \pm 0.23 \pm 0.03$
Belle [8]	$0.05 \pm 0.14 \pm 0.05$	$0.33 \pm 0.35 \pm 0.08$
Average [5]	0.14 ± 0.11	0.38 ± 0.19

Table II: CP-averaged branching ratios and CP rate asymmetries for $B \to K\pi$ decays and $B^+ \to \pi^+\pi^0$, based on averages in Ref. [5].

Mode	Branching ratio (10^{-6})	A_{CP}
$B^0 \to K^+\pi^-$	19.4 ± 0.6	-0.097 ± 0.012
$B^0 \to K^0\pi^0$	9.8 ± 0.6	-0.14 ± 0.11
$B^+ \to K^0\pi^+$	23.1 ± 1.0	0.009 ± 0.025
$B^+ \to K^+\pi^0$	12.9 ± 0.6	0.050 ± 0.025
$B^+ \to \pi^+\pi^0$	$5.59^{+0.41}_{-0.40}$	0.06 ± 0.05

The current status of measurements of $C_{K\pi}$ and $S_{K\pi}$ is summarized in Table I. The value of $C_{K\pi}$ is consistent with the pure-penguin value of zero, while that of $S_{K\pi}$ is 1.6σ below the pure-penguin value of $\sin 2\beta = 0.681 \pm 0.025$.

A sum rule for direct CP asymmetries in $B \to K\pi$ decays has been derived purely on the basis of the $\Delta I = 0$ property of the dominant penguin amplitude, using an isospin quadrangle relation among the four $B \to K\pi$ decay amplitudes which depend also on two $\Delta I = 1$ amplitudes [9, 10]:

$$A(B^0 \to K^+\pi^-) + \sqrt{2}A(B^0 \to K^0\pi^0) = A(B^+ \to K^0\pi^+) + \sqrt{2}A(B^+ \to K^+\pi^0) \ . \tag{2}$$

In its most precise form the sum rule relates the four CP rate differences [11],

$$\Delta(K^+\pi^-) + \Delta(K^0\pi^+) = 2\Delta(K^+\pi^0) + 2\Delta(K^0\pi^0) \ , \tag{3}$$

where one defines

$$\Delta(f) \equiv \Gamma(\bar{B} \to \bar{f}) - \Gamma(B \to f) \ . \tag{4}$$

This sum rule includes interference terms of the dominant penguin amplitude with all small non-penguin contributions. A few very small quadratic terms representing interference of tree and electroweak penguin amplitudes vanish in the SU(3) and heavy quark limits [11].

Using the decay branching ratios and CP asymmetries summarized in Table II [5] and the known lifetime ratio $\tau(B^+)/\tau(B^0) = 1.071 \pm 0.009$ [5], one can use this relation to solve for the least-well-known quantity $\Delta(K^0\pi^0)$, implying

$$A_{CP}(K^0\pi^0) = -0.148 \pm 0.044 \ . \tag{5}$$

The error on the right-hand-side is dominated by the current experimental errors in $A_{CP}(K^0\pi^+)$ and $A_{CP}(K^+\pi^0)$. The prediction (5) following from (3) involves a smaller theoretical uncertainty at a percent level from quadratic terms describing the interference of small non-penguin amplitudes. Verification of this prediction would provide evidence that non-penguin amplitudes behave as expected in the Standard Model. [If one uses the corresponding sum rule for CP asymmetries,

$$A_{CP}(K^+\pi^-) + A_{CP}(K^0\pi^+) = A_{CP}(K^+\pi^0) + A_{CP}(K^0\pi^0) \ , \tag{6}$$

one predicts $A_{CP}(K^0\pi^0) = -0.138 \pm 0.037$. Using this relation with $A_{CP}(K^0\pi^+) = 0$, as expected since $B^+ \to K^0\pi^+$ should be dominated by a penguin amplitude with only a very small annihilation contribution [12], one predicts $A_{CP}(K^0\pi^0) = -0.147 \pm 0.028$.]

Non-penguin amplitudes are generally agreed to increase $S_{K\pi}$ from its pure-penguin value of $\sin 2\beta = 0.681 \pm 0.025$ by a modest amount, generally to 0.8 or below [13–16]. Model-independent bounds using flavor SU(3) [17, 18] also favor at most a deviation of 0.2 from the pure-penguin value. An exception is noted in the treatments of Refs. [19] and [20], and most recently in Ref. [6], where a relation between $C_{K\pi}$ and $S_{K\pi}$ was studied implying a value $S_{K\pi} = 0.99$ for the central value measured for $C_{K\pi}$. A geometrical construction is performed which illustrates the way in which such a large value arises.

An aspect of the prediction of $S_{K\pi} \simeq 0.99$ which has not been sufficiently stressed is its extreme sensitivity to the branching ratio $\mathcal{B}(B^0 \to K^0\pi^0)$. In the present Letter we analyze the sensitivity of $S_{K\pi}$ to this and other observables within the Standard Model, and highlight those measurements which would shed light on the presence of new physics. In order to restrict the range allowed for $S_{K\pi}$ in the Standard Model one needs certain information about strong phases. Theoretical calculations of strong phases in $B \to K\pi$ based on $1/m_b$ expansions are known to fail, most likely because of long distance charming penguin contributions [21, 22]. We propose to obtain the necessary information about strong phases directly from experiments. Somewhat different but not completely independent arguments were presented in Ref. [6].

The $B \to K\pi$ amplitudes may be decomposed into contributions from various amplitudes as follows [23, 24]:

$$\begin{aligned} A_{+-} &\equiv A(B^0 \to K^+\pi^-) = -(p+t) \quad , \\ A_{00} &\equiv \sqrt{2}A(B^0 \to K^0\pi^0) = p - c \quad , \\ A_{0+} &\equiv A(B^+ \to K^0\pi^+) = p + A \quad , \\ A_{+0} &\equiv \sqrt{2}A(B^+ \to K^+\pi^0) = -(p+t+c+A) \quad , \end{aligned} \tag{7}$$

$$t \equiv T + P^C_{\rm EW} \ , \quad c \equiv C + P_{\rm EW} \ , \quad p \equiv P - \frac{1}{3}P^C_{\rm EW} \quad . \tag{8}$$

The terms T, C and A represent color-favored and color-suppressed tree amplitudes and a small annihilation term, while P stands for a gluonic penguin amplitude. Color-favored and color-suppressed electroweak penguin amplitudes are represented by $P_{\rm EW}$ and $P^C_{\rm EW}$. The sums of the first two and last two amplitudes in Eq. (7) are equal [see Eq. (2)] and both correspond to an amplitude $A_{3/2}$ for a $K\pi$ state with isospin $I_{K\pi} = 3/2$ [9, 10]:

$$A(B^0 \to K^+\pi^-) + \sqrt{2}A(B^0 \to K^0\pi^0) = A(B^+ \to K^0\pi^+) + \sqrt{2}A(B^+ \to K^+\pi^0)$$

$$= -(t + c) = -(T + C + P^C_{\rm EW} + P_{\rm EW}) = A_{3/2} \ . \tag{9}$$

The contribution $-(T+C)$ to $A_{3/2}$ has a magnitude which can be obtained from the decay $B^+ \to \pi^+\pi^0$ via flavor SU(3) [25],

$$|T + C| = \sqrt{2}\frac{V_{us}}{V_{ud}}\frac{f_K}{f_\pi}\xi_{T+C}|A(B^+ \to \pi^+\pi^0)| \ . \tag{10}$$

SU(3) breaking in this amplitude is often assumed to be given by the factor $f_K/f_\pi = 1.193 \pm 0.006$ [26]. Here we introduce a parameter $\xi_{T+C} = 1.0 \pm 0.2$ which represents an uncertainty in this factor. The weak phase of $T + C$ is $\text{Arg}(V^*_{ub}V_{us}) = \gamma$, where $\gamma = (65 \pm 10)^\circ$ [27]. We take its strong phase to be zero by convention. All other strong phases will be taken in the range $(-\pi, \pi)$. The penguin amplitude P dominating $B \to K\pi$ decays carries the weak phase $\text{Arg}(V^*_{tb}V_{ts}) = \pi$. Its strong phase relative to that of $T + C$ will be denoted $-\delta_c$ [28]. Thus

$$T + C = |T + C|e^{i\gamma} \ , \qquad\qquad P = -|P|e^{-i\delta_c} \ . \tag{11}$$

The electroweak penguin contribution $P^C_{\rm EW} + P_{\rm EW}$ was shown in Refs. [29] and [30] to have the same strong phase as $T + C$ in the SU(3) symmetry limit. In this limit the ratio of these two amplitudes is given numerically in terms of ratios of CKM factors and Wilson coefficients, $(P_{\rm EW} + P^C_{\rm EW})/(T + C) = -0.66\xi_{EW}e^{-i\gamma}$. The parameter ξ_{EW} includes an uncertainty from SU(3) breaking, which we will take as $\xi_{EW} = 1.0 \pm 0.2$, and a smaller uncertainty from CKM factors. We neglect a potential small strong phase of ξ_{EW} which has a negligible effect on our analysis below. Thus we have an amplitude triangle relation,

$$A_{00} + A_{+-} = A_{3/2} = -|T + C|\left(e^{i\gamma} - 0.66\xi_{EW}\right) \ , \tag{12}$$

and a similar relation for the CP-conjugate amplitudes in which the sign of γ is reversed.

In order to visualize the geometric construction of the triangle (12) and its CP-conjugate, as proposed in Ref. [6] but with realistic quantities including the restricted range (5) for $A_{CP}(K^0\pi^0)$, we express all branching ratios in units of 10^{-6}, and take amplitudes as their square roots. (We first divide B^+ branching ratios by the lifetime ratio $\tau(B^+)/\tau(B^0) = 1.071 \pm 0.009$ [5] to compare them with B^0 branching ratios.) The central values of $|T+C|$ for $\xi_{T+C} = 1$ and the squares $|A_{ij}|^2$ and $|\bar{A}_{ij}|^2$, based on central values of the rates and CP asymmetries in Table II, are

$$\begin{aligned}
|T + C| &= 0.900 \ , \\
|A_{00}|^2 &= 2(9.8)(1 + 0.14) = 22.3 \ , \\
|A_{+-}|^2 &= (19.4)(1 + 0.097) = 21.3 \ , \\
|\bar{A}_{00}|^2 &= 2(9.8)(1 - 0.14) = 16.9 \ , \\
|\bar{A}_{-+}|^2 &= (19.4)(1 - 0.097) = 17.5 \ .
\end{aligned} \tag{13}$$

Solutions for the amplitude triangle (12) and its CP-conjugate may be obtained analytically by solving simple quadratic equations for the central values of the parameters which fix $A_{3/2}$ in (12), $\xi_{EW} = 1$, $\gamma = 65^\circ$. The quadratic equation for each triangle has

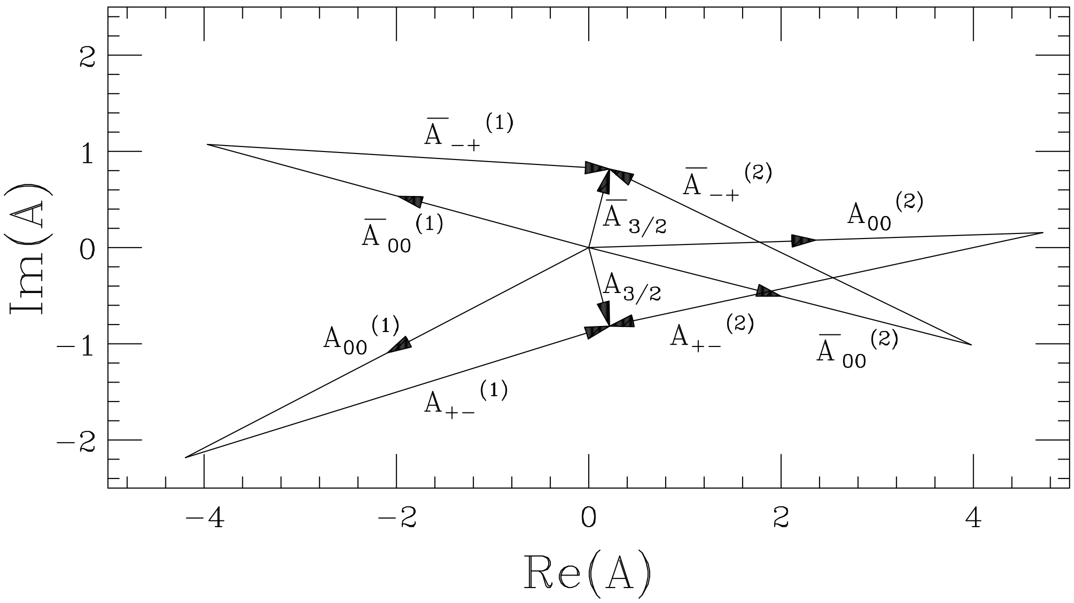

Figure 1: Triangles relating amplitudes for $B^0 \to K^0\pi^0$ and $B^0 \to K^+\pi^-$ to the amplitude $A_{3/2}$, and triangles for the corresponding charge-conjugate processes.

two solutions, which can be visualized by flipping the triangle around the side $A_{3/2}$ or $\bar{A}_{3/2}$ which is kept fixed. One thus obtains a total of $2 \times 2 = 4$ solutions, of which two are illustrated in Fig. 1. The other two solutions correspond to flipping one triangle but not the other.

We have chosen to express the triangles with A_{00} or $\bar{A}_{00}$ emanating from the origin, in order to illustrate the relative phase of A_{00} and $\bar{A}_{00}$ which will be important in the evaluation of $S_{K\pi}$. This relative phase vanishes in the limit of pure penguin dominance and is expected to be smaller than $\pi/2$ when including small color-suppressed tree and electroweak penguin contributions in A_{00}. This feature holds true for the two illustrated solutions but excludes the two solutions with one triangle flipped, for which the relative phase between A_{00} and $\bar{A}_{00}$ is larger than $\pi/2$.

The expected value of $S_{K\pi}$ is related to the magnitudes and phases of A_{00} and $\bar{A}_{00}$ in the following manner:

$$S_{K\pi} = \frac{2|A_{00}\bar{A}_{00}|}{|A_{00}|^2 + |\bar{A}_{00}|^2} \sin(2\beta + \phi_{00}) \ . \tag{14}$$

The correction $\phi_{00} \equiv \mathrm{Arg}(A_{00}\bar{A}^*_{00})$ to 2β is found to be positive for both of the displayed solutions. It is quite large, $\phi_{00} = 42.6^\circ$ corresponding to $S_{K\pi} = 0.99$, for the solution (1) with negative real values of the amplitudes A_{00} and $\bar{A}_{00}$ and smaller, $\phi_{00} = 16.1^\circ$ corresponding to $S_{K\pi} = 0.85$, for the solution (2) with positive real values. Since A_{00} is dominated by the penguin amplitude, $P = -|P|\exp(-i\delta_c)$, solution (1) corresponds to $\cos\delta_c > 0$ ($|\delta_c| < \pi/2$) while solution (2) involves $\cos\delta_c < 0$ ($|\delta_c| > \pi/2$).

In order to exclude solution (2) one would have to show unambiguously that $\cos\delta_c > 0$ or $|\delta_c| < \pi/2$, where δ_c is the strong phase difference between $T + C$ and P. A most

direct proof for $\cos\delta_c > 0$ would need an observation of destructive interference between P and $T + C$ in the CP-averaged decay rate of $B^+ \to K^+\pi^0$ normalized by that of $B^+ \to K^0\pi^+$. However, this interference is cancelled by constructive interference of P and $P_{EW} + P^C_{EW}$ [31]. Arguments for small strong phase differences including δ_c have been presented in studies of $B \to K\pi$ and $B \to \pi\pi$ based on a heavy quark expansion [32]. These arguments failed, however, when predicting a very small phase $\mathrm{Arg}(C/T)$. This would imply $A_{CP}(K^+\pi^0) < A_{CP}(K^+\pi^-)$, contrary to the two asymmetries quoted in Table II, which show that this phase is not very small and must be negative (see argument below [31].) A small value of δ_c ($|\delta_c| < 30°$) was obtained in global flavor SU(3) fits to decay rates and CP asymmetries measured for $B \to K\pi$ and $B \to \pi\pi$ [13, 33]. Within these fits it is difficult to pinpoint a small subset of $B \to K\pi$ measurements forcing a small value for δ_c. The purpose of the subsequent discussion is to prove $\cos\delta_c > 0$ using a series of arguments based on specific measurements, stressing the minimal use of untested assumptions about flavor SU(3).

A strong phase which is more directly accessible to experiment than δ_c is δ, the strong phase of T relative to that of P. This phase occurs in the amplitude for $B^0 \to K^+\pi^-$. Its cosine term appears in the ratio R of CP-averaged decay rates for this process and $B^+ \to K^0\pi^+$ [34, 35]. Neglecting P^C_{EW} and A terms in these amplitudes, one would expect R to be smaller than one for $\cos\delta > 0$ and larger than one for $\cos\delta < 0$. The current value $R = 0.899 \pm 0.048$, obtained from branching ratios in Table II and the above-mentioned ratio of B^+ and B^0 lifetimes, favors $\cos\delta > 0$ over $\cos\delta < 0$. This evidence is statistically limited and may suffer from P^C_{EW} corrections in $B^0 \to K^+\pi^-$. The negative asymmetry $A_{CP}(K^+\pi^-) = -0.097 \pm 0.012$ proves unambiguously that δ is positive.

An argument proving $|\delta| < \pi/2$ unambiguously is based on the time-dependent CP asymmetry parameter $S_{\pi^+\pi^-}$ in $B^0 \to \pi^+\pi^-$. Assuming flavor SU(3), the ratio of penguin and tree amplitudes and their relative phase are equal in this process to those in $B^0 \to K^+\pi^-$, up to CKM factors defining the ratios of amplitudes. Neglecting small W-exchange and penguin annihilation contributions (the resulting systematic uncertainty introduced by this approximation is taken as part of an uncertainty due to SU(3) breaking mentioned below), one has [36]

$$S_{\pi^+\pi^-} = \frac{\sin 2\alpha + 2r\cos\delta\sin(\beta-\alpha) - r^2\sin 2\beta}{1 - 2r\cos\delta\cos(\beta+\alpha) + r^2} \ , \tag{15}$$

where $\alpha = \pi - \beta - \gamma$ and r is the ratio of penguin and tree amplitudes in $B^0 \to \pi^+\pi^-$. In the absence of a penguin amplitude one has $S_{\pi^+\pi^-} = \sin 2\alpha$, and to first order in the ratio r one finds [37]

$$S_{\pi^+\pi^-} = \sin 2\alpha + 2r\cos\delta\sin(\beta+\alpha)\cos 2\alpha \ . \tag{16}$$

BaBar [38] and Belle [39] find the same value for this quantity; the average is large and negative [5], $S_{\pi^+\pi^-} = -0.61 \pm 0.08$. Since $\alpha = \pi - \beta - \gamma \simeq \pi/2$ [27] one has $\sin 2\alpha \simeq 0$ and $\cos 2\alpha \simeq -1$, while $\sin(\beta + \alpha) > 0$, which implies $\cos\delta > 0$.

A detailed analysis using the exact expression (15) and measurements of $S_{\pi^+\pi^-}$ and a second asymmetry $C_{\pi^+\pi^-} \equiv -A_{CP}(\pi^+\pi^-)$ confirmed this conclusion obtaining

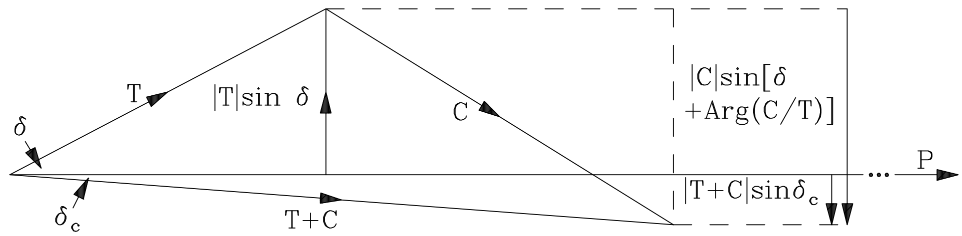

Figure 2: Illustration of relative strong phases of T, C, and P in $B \to K\pi$ decays and the construction leading to Eq. (17). Here $\delta = \text{Arg}(T/P)$; $\delta_c = \text{Arg}[(T+C)/P]$.

a value $\delta = (33 \pm 7^{+8}_{-10})^\circ$ [37]. The first error is experimental, while the second is associated with a systematic uncertainty in flavor-SU(3) breaking. The positive sign of δ, following from the negative averaged $C_{\pi^+\pi^-}$, agrees with the negative value of the measured $A_{CP}(K^+\pi^-)$. The two CP rate asymmetries are equal within experimental errors and have opposite signs [40, 41]. Expressed in units of 10^{-6} they are $\Delta(K^+\pi^-) = -1.88 \pm 0.24 = -\Delta(\pi^+\pi^-) = -1.96 \pm 0.37$ [5]. This confirms the flavor SU(3) assumption for equal ratios of penguin and tree amplitudes and equal relative strong phases in these two processes. A difference of 180° between the two phases, which would not affect the equality of CP rate asymmetries, is extremely unlikely. The property $|\delta| < \pi/2$ implies constructive (destructive) interference between T and P in the CP averaged rate for $B^0 \to \pi^+\pi^-$ ($B^0 \to K^+\pi^-$).

In order to constrain δ_c (the strong phase difference between $T+C$ and P), using the above range for δ (the strong phase difference between T and P), one needs information about the strong phase of the ratio C/T. The observation $A_{CP}(K^+\pi^0) > A_{CP}(K^+\pi^-)$ implies that $\text{Arg}(C/T)$ is negative and larger in magnitude than δ [31]. A simple proof of this behavior, for terms in the two asymmetries which are linear in $|T+C|/|P|$ and $|T|/|P|$, respectively, follows from the geometrical identity

$$|T+C| \sin\delta_c = |T| \sin\delta + |C| \sin[\delta + \text{Arg}(C/T)] \tag{17}$$

illustrated in Fig. 2. The amplitudes $T + C$ interfere constructively in $B^+ \to \pi^+\pi^0$. This follows from the observation that $2\mathcal{B}(B^+ \to \pi^+\pi^0) > \mathcal{B}(B^0 \to \pi^+\pi^-)$ [5], and the above-mentioned constructive interference of T and P in $B^0 \to \pi^+\pi^-$. Thus $-\pi/2 < \text{Arg}(C/T) < -\delta < 0$ which implies geometrically $-\pi/2 < \delta_c < \delta < \pi/2$, without making any assumption about the magnitude $|C/T|$. This concludes the proof of $\cos\delta_c > 0$ which excludes solution (2) in Fig. 1.

It is the large value of $\phi_{00} \equiv \text{Arg}(A_{00}\bar{A}^*_{00})$ in solution (1) in Fig. 1 which is thus responsible for boosting the expected value of $S_{K\pi}$ from its penguin-dominated value of $\sin 2\beta \simeq 0.68$ to a value very close to 1. We now explore the sensitivity of this effect to small changes in experimental inputs.

We find the greatest sensitivity of $S_{K\pi}$ is to variations of the branching ratio $\text{B}(K^0\pi^0) \equiv \mathcal{B}(B^0 \to K^0\pi^0)$. In Fig. 3(a) we plot ϕ_{00} and $S_{K\pi}$ versus $\text{B}(K^0\pi^0)$ for nominal values of the parameters noted in the text. We note that $S_{K\pi}$ drops from a value of 0.99 at the central value of $\text{B}(K^0\pi^0)$ to 0.91 and 0.72 at -1σ and -2σ below the central value.

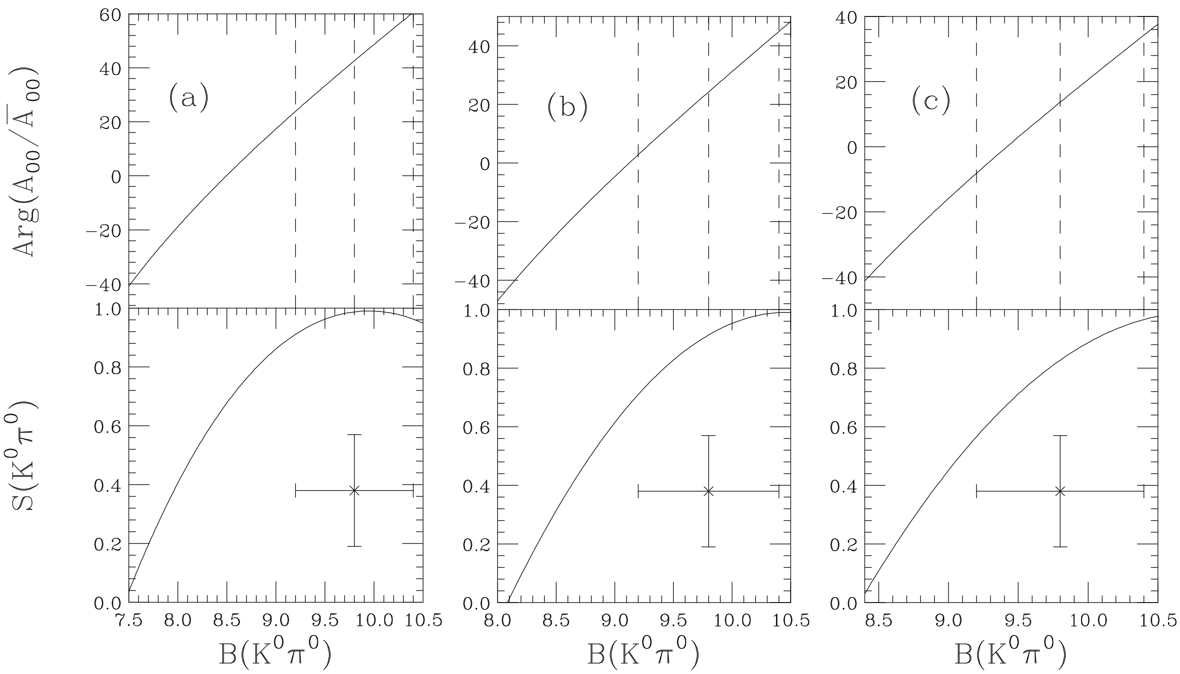

Figure 3: Dependence of $\text{Arg}(A_{00}/\bar{A}_{00})$ and $S_{K\pi}$ on $\text{B}(K^0\pi^0) \equiv \mathcal{B}(B^0 \to K^0\pi^0)$. Vertical dashed lines in top panel show central value and $\pm 1\sigma$ errors of $\text{B}(K^0\pi^0)$. The plotted point on the lower panels shows the experimental values. (a) All parameters as in text; (b) same as (a), but $\gamma = 55°$; (c) same as (b), but $\mathcal{B}(B^0 \to K^+\pi^-) = 20 \times 10^{-6}$.

Table III: Comparison of sensitivity of $\phi_{00} \equiv \text{Arg}(A_{00}\bar{A}^*_{00})$ (in degrees) and $S_{K\pi}$ to various parameters.

Parameter	-1σ		$+1\sigma$	
	ϕ_{00}	$S_{K\pi}$	ϕ_{00}	$S_{K\pi}$
$\mathcal{B}(B^0 \to K^0\pi^0)$	23.9	0.911	60.6	0.963
γ	24.3	0.913	59.4	0.967
$\mathcal{B}(B^0 \to K^+\pi^-)$	52.0	0.986	33.3	0.962
ξ_{T+C}	41.0	0.985	44.4	0.989
ξ_{EW}	26.3	0.926	58.0	0.972

We next vary γ within its 1σ limits to 55° [Fig. 3(b)]. The experimental values become considerably more compatible with the Standard Model predictions, and even more so if $\mathcal{B}(B^0 \to K^+\pi^-)$ is increased by 1σ to 20×10^{-6} [Fig. 3(c)]. In Figs. 3 the quantity ϕ_{00} is more sensitive than $S_{K\pi}$ to variations in $\mathcal{B}(B^0 \to K^0\pi^0)$, γ, and $\mathcal{B}(B^0 \to K^+\pi^-)$. For the central value of ϕ_{00}, $S_{K\pi}$ is very close to its maximum value, so it is only for considerably lower values of ϕ_{00} that $S_{K\pi}$ becomes sensitive to these parameters.

In Table III we summarize the effects on ϕ_{00} and $S_{K\pi}$ of varying $\mathcal{B}(B^0 \to K^0\pi^0)$, γ, and $\mathcal{B}(B^0 \to K^+\pi^-)$ by $\pm1\sigma$ around their central values. (See Table II; we are taking $\gamma = (65 \pm 10)^\circ$.) A possible effect combining these three errors is seen in Fig. 3(c). We also include the effects of $\pm1\sigma$ variations of $\xi_{T+C} = 1.0 \pm 0.2$ and $\xi_{EW} = 1.0 \pm 0.2$. For nominal values of the parameters, one has $\phi_{00} = 42.6^\circ$ and $S_{K\pi} = 0.987$. Table III indicates the greatest sensitivity of ϕ_{00} to $\mathcal{B}(B^0 \to K^0\pi^0)$, followed by γ and ξ_{EW}. There is relatively little sensitivity to ξ_{T+C}.

Other variations are found to have a negligible effect on $S_{K\pi}$. This includes the asymmetry $A_{CP}(B^0 \to K^+\pi^-)$, which involves a very small experimental error, and $A_{CP}(B^0 \to K^0\pi^0) \equiv -C_{K\pi}$, which is predicted in (5) with a small uncertainty. A large variation in this asymmetry would in any case have little effect on $S_{K\pi}$, as a geometric construction similar to that in Fig. 1 illustrates. The phases of A_{00} and $\bar{A}_{00}$ are found to shift nearly together, so that the correction to $\sin 2\beta$ in Eq. (14) changes very little. This insensitivity to $C_{K\pi}$ is displayed for the favored $S_{K\pi}$ solution in Ref. [6], where $C_{K\pi}$ is left unconstrained disregarding the sum rule (3).

Thus the possibility that the above calculation of $S_{K\pi}$ in the Standard Model differs both from its penguin-dominated value of $\sin 2\beta \simeq 0.68$ and from the data remains intriguing. However, for it to become a robust conclusion about the presence of new physics, accuracies of measurements of the B^0 branching ratios to $K^0\pi^0$ and $K^+\pi^-$ and of the CKM angle γ need to be improved. We look forward to such advances in future data, and to more precise measurements of the two asymmetries $C_{K\pi}$ and $S_{K\pi}$ in $B^0 \to K^0\pi^0$.

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Note added: The measurements of $C_{K\pi}$ and $S_{K\pi}$ given in Table I have been updated very recently by the BaBar and Belle collaborations. New results and their averages are summarized in Table IV. The averaged value of $C_{K\pi}$ agrees with the prediction

Table IV: Updated measurements of $C_{K\pi}$ and $S_{K\pi}$.

Ref.	$C_{K\pi}$	$S_{K\pi}$
BaBar [42]	$0.13 \pm 0.13 \pm 0.03$	$0.55 \pm 0.20 \pm 0.03$
Belle [43]	$-0.14 \pm 0.13 \pm 0.06$	$0.67 \pm 0.31 \pm 0.08$
Average	0.00 ± 0.10	0.58 ± 0.17

(5) within 1.4σ, while $S_{K\pi}$ is now consistent with $\sin 2\beta$ and somewhat larger values. Recent updates by BaBar of the branching ratio for $B^0 \to K^0\pi^0$ and the CP asymmetry in $B^0 \to K^+\pi^-$ [44] do not affect significantly the corresponding two averaged values in Table II.

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